Class Test #5 | Student # Solution | Time: 15 Mins.
ME 449 : Composite Materials | Student Name: to CT #4

Find the residual stresses at the top, mid, and bottom surfaces of the 90° ply in a $[0/90]_s$ carbon/epoxy laminate subjected to a temperature change of −100°C. Each lamina is 0.127 mm thick. Use the following properties of the unidirectional carbon/epoxy lamina:

$$[Q]=\begin{bmatrix}141 & 3 & 0\\ 3 & 10 & 0\\ 0 & 0 & 5\end{bmatrix}\text{GPa};\ \alpha_1=-0.9\times10^{-6}/°\text{C};\ \alpha_2=27\times10^{-6}/°\text{C}$$ and total strain is zero.

$\Delta T = -100^{\circ}C$

$$\begin{bmatrix}\sigma_x^R\\ \sigma_y^R\\ \tau_{xy}^R\end{bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\left\{\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix}+z\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix}-\begin{bmatrix}\varepsilon_x^*\\ \varepsilon_y^*\\ \gamma_{xy}^*\end{bmatrix}\right\}$$

$$=-\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^*\\ \varepsilon_y^*\\ \gamma_{xy}^*\end{bmatrix} \quad \text{(1)}$$

$$\begin{bmatrix}\varepsilon_x^*\\ \varepsilon_y^*\\ \gamma_{xy}^*\end{bmatrix}_{90}=\begin{bmatrix}c^2 & s^2 & -cs\\ s^2 & c^2 & cs\\ 2cs & -2cs & c^2-s^2\end{bmatrix}\begin{bmatrix}\varepsilon_1^*\\ \varepsilon_2^*\\ 0\end{bmatrix};\quad \begin{bmatrix}\varepsilon_1^*\\ \varepsilon_2^*\\ \gamma_{12}^*\end{bmatrix}=\Delta T\begin{bmatrix}\alpha_1\\ \alpha_2\\ 0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}0.9\times10^{-4}\\ -27\times10^{-4}\\ 0\end{bmatrix} \qquad =-100\begin{bmatrix}-0.9\times10^{-6}\\ 27\times10^{-6}\\ 0\end{bmatrix}$$

$$=\begin{bmatrix}-27\times10^{-4}\\ 0.9\times10^{-4}\\ 0\end{bmatrix}\ m/m \qquad =\begin{bmatrix}0.9\times10^{-4}\\ -27\times10^{-4}\\ 0\end{bmatrix}$$

$$\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_{90}=\begin{bmatrix}10 & 3 & 0\\ 3 & 141 & 0\\ 0 & 0 & 5\end{bmatrix}$$

$$\begin{bmatrix}\sigma_x^R\\ \sigma_y^R\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}10 & 3 & 0\\ 3 & 141 & 0\\ 0 & 0 & 5\end{bmatrix}\begin{bmatrix}-27\times10^{-4}\\ 0.9\times10^{-4}\\ 0\end{bmatrix}=\begin{bmatrix}-0.02673\\ 4.59\times10^{-3}\\ 0\end{bmatrix}\ GPa$$

Ans°